It is frequently of interest to compare two samples without any assumptions about the population distribution, and SimFIT provides an interface to conduct such nonparametric tests for equality of the median and dispersion, i.e. the variance, with two such samples.

Open the main SimFIT menu, choose [A/Z], then select the SimFIT nonparametric test program **rstest**, and run the Median, Mood, and David tests using the following default data

X-values	6	9	12	4	10	11
Y-values	8	1	3	7	2	5

leading to these results.

Median, Mood and David tests number 1	
Current data sets X and Y are:	
G08BAF.TF1: Mood-David tests for equal dispersions	
Number of X-values	6
G08BAF.TF2: Mood-David tests for equal dispersions	
Number of Y-values	6
Results for the median test:	
H_0: medians are the same	
Number of X-scores $<$ pooled median	2
Number of Y-scores $<$ pooled median	4
Probability under H_0	0.2835
Results for the Mood test	
H_0: dispersions are equal	
H_1: X-dispersion $>$ Y-dispersion	
H_2: X-dispersion $<$ Y-dispersion	
The Mood test statistic	75.50
Probability under H_0	0.8339
Probability under H_1	0.4170
Probability under H_2	0.5830
Results for the David test	
H_0: dispersions are equal	
H_1: X-dispersion $>$ Y-dispersion	
H_2: X-dispersion $<$ Y-dispersion	
The David test statistic	9.467
Probability under H_0	0.3972
Probability under H_1	0.8014
Probability under H_2	0.1986

As usual with SimFIT, all three results are given for convenience, but with the understanding that either only one pre-decided test is to be used, or that the Bonferroni correction will be employed if more than one test result is to be considered.

These tests all start by forming a pooled sample, then calculating the overall median M of the pooled sample and considering various functions of the ranks r_i within this pooled sample. It is not surprising that with such small samples no significant differences were detected in this case.

However, to better understand what these tests do, you should now use test files `g08acf.tf1` and `g08acf.tf2`, which have larger and more distinct samples and lead to the following results.

Median, Mood and David tests number 2		
Current data sets X and Y are:		
G08ACF.TF1: the median test		
Number of X-values	16	
G08ACF.TF2: the median test		
Number of Y-values	23	
	7	
Results for the median test:		
H_0: medians are the same		
Number of X-scores < pooled median	13	
Number of Y-scores < pooled median	6	
Probability under H_0	0.0009	Reject H_0 at 1% significance level
Results for the Mood test		
H_0: dispersions are equal		
H_1: X-dispersion > Y-dispersion		
H_2: X-dispersion < Y-dispersion		
The Mood test statistic	1947	
Probability under H_0	0.8200	
Probability under H_1	0.5900	
Probability under H_2	0.4100	
Results for the David test		
H_0: dispersions are equal		
H_1: X-dispersion > Y-dispersion		
H_2: X-dispersion < Y-dispersion		
The David test statistic	69.77	
Probability under H_0	0.0130	Reject H_0 at 5% significance level
Probability under H_1	0.9935	
Probability under H_2	0.0065	Reject H_0 at 1% significance level

The calculations used to perform these tests will now be outlined.

The Median test

If there are n observations overall, with individual sample sizes n_x and n_y so that $n = n_x + n_y$, then the data can be expressed as a 2 by 2 contingency table with frequencies

$$
\begin{aligned}
f_{11} &= \text{ Number of } X \leq M \\
f_{21} &= n_x - f_{11} \\
f_{12} &= \text{ Number of } Y \leq M \\
f_{22} &= n_y - f_{12}
\end{aligned}
$$

then a chi-square test, or with small samples ($n \leq 100$) a Fisher exact test, is carried out. The analysis for these data leads to the following table of results when a contingency table analysis is performed using SimFIT, but displaying only the most important results.

Fisher exact test		
Observed	Rearranged so r_1 = smallest marginal, $c_2 \geq c_1$	
13 6	13 3	
3 17	6 17	
$p(13)$	0.000820	$p(*)$, observed frequencies
$p(14)$	0.000059	
$p(15)$	0.000002	
$p(16)$	0.000000	
P_sum3	0.000881	sum of all $p(r)$ for $r \geq 13$

Of course, it is obvious from the way the two data sets are partitioned by the overall median M in this contingency table that the Y values tend to be larger than the X values, and the Fisher exact probability confirms this. Note that, in order to calculate the significance level for this table, the Fisher exact test must not only consider the probability of the given table $p(*)$ but must add the sum of probabilities for the more extreme tables, i.e., with f_{11} equal to 14, 15, and 16.

Mood's test

This assumes that the two samples have the same mean so that

$$W = \sum_{i=1}^{n_x} \left(r_i - \frac{n+1}{2}\right)^2,$$

which is the sum of squares of deviations from the average rank in the pooled sample, is approximately normally distributed for large n. The test statistic is

$$z = \frac{W - n_x(n^2 - 1)/12}{\sqrt{n_x n_y (n+1)(n^2 - 4)/180}}.$$

This test suffers from the disadvantage that is assumes equal means for the two samples and, if this is not justified, it can lead to inflated values for W.

David's test

This test uses the mean rank

$$\bar{r} = \sum_{i=1}^{n_x} r_i / n_x$$

to reduce the effect of the assumption of equal means in Mood's test by calculating

$$V = \frac{1}{n_x - 1} \sum_{i=1}^{n_x} (r_i - \bar{r})^2,$$

and V is also approximately normally distributed for large n. The test statistic is

$$z = \frac{V - n(n+1)/12}{\sqrt{n n_y (n+1)(3(n+1)(n_x+1) - n n_x)/360 n_x (n_x - 1)}}.$$

Note that it is not the values of W or V alone that determine the significance level for these dispersion tests, but the z statistics calculated from them as defined above. It is often recommended that David's test is more discerning than Mood's test, which seems to be the case with these data.