Tutorials and worked examples for simulation,
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http://www.simfit.org.uk

The situation envisaged is where a user has a set counts of nonnegative integers and wishes to see if the frequencies observed are consistent with a Poisson distribution with parameter λ. In SIMFIT this can be done using a chi-square test, a dispersion test, a Fisher exact test, or a Kolmogorov-Smirnov test.

From the SIMFIT main menu choose `[A/Z]`, then program **binomial**, and select the option to test for a Poisson distribution. The most widely used test is a chi-square test so this is done first, choosing to use the sample estimate of $\hat{\lambda} = 1.1$ instead of the current fixed value of 2, and opting for a minimum of 5 counts per bin, leading to the next results.

Chi-square test for $P(\lambda)$ with $\lambda = 1.1$

H_0: Poisson distribution for data with title:		
Test file `Poisson.tf1`: 40 random numbers		
Sample estimate used in chi-square test		
Sample estimate for λ	1.100	
Lower 95% confidence limit	0.7993	
Upper 95% confidence limit	1.477	
Mean of x-values	1.100	
Variance of x-values	0.8103	
Standard deviation of x	0.9001	
Mean using fixed λ	2.000	
Poisson dispersion value D	28.73	
$P(\chi^2 \geq D)$	0.8863	
Number of partitions (bins) used	3	
Number of degrees of freedom	1	
Chi-square test statistic C	5.857	
$P(\chi^2 \geq C)$	0.0155	*Reject H_0 at 5% level*
Upper tail 5% critical point	3.841	
Upper tail 1% critical point	6.635	

Now it should be emphasized that the chi-square test is an approximate test, as the test statistic only becomes asymptotic to a chi-square variable with large samples. Further, if any cells have a small frequency, say < 5, it is usually recommended to combine adjacent bins until this is the case. That is why choosing a minimum frequency of 5 resulted in only 3 bins. If the test is now repeated on the same data but choosing a minimum frequency of 3 the next results are obtained.

Number of partitions (bins) used	4	
Number of degrees of freedom	2	
Chi-square test statistic C	5.858	
$P(\chi^2 \geq C)$	0.0535	*Consider accepting H_0*
Upper tail 5% critical point	5.991	
Upper tail 1% critical point	9.210	

It should be noted that by simply changing the number of bins from 3 to 4 a rejection ($p = 0.0155$) becomes an acceptance ($p = 0.053$), which should serve to emphasize how the outcome of such chi-square tests depend on the number of bins.

A classical example is the famous data collected by von Bortkiewicz for 0, 1, 2, 3, or 4 deaths per year by horse kick in the Prussian cavalry during the period 1875 - 1894 for 10 groups, as this is a typical example of using the Poisson distribution to model rare events.

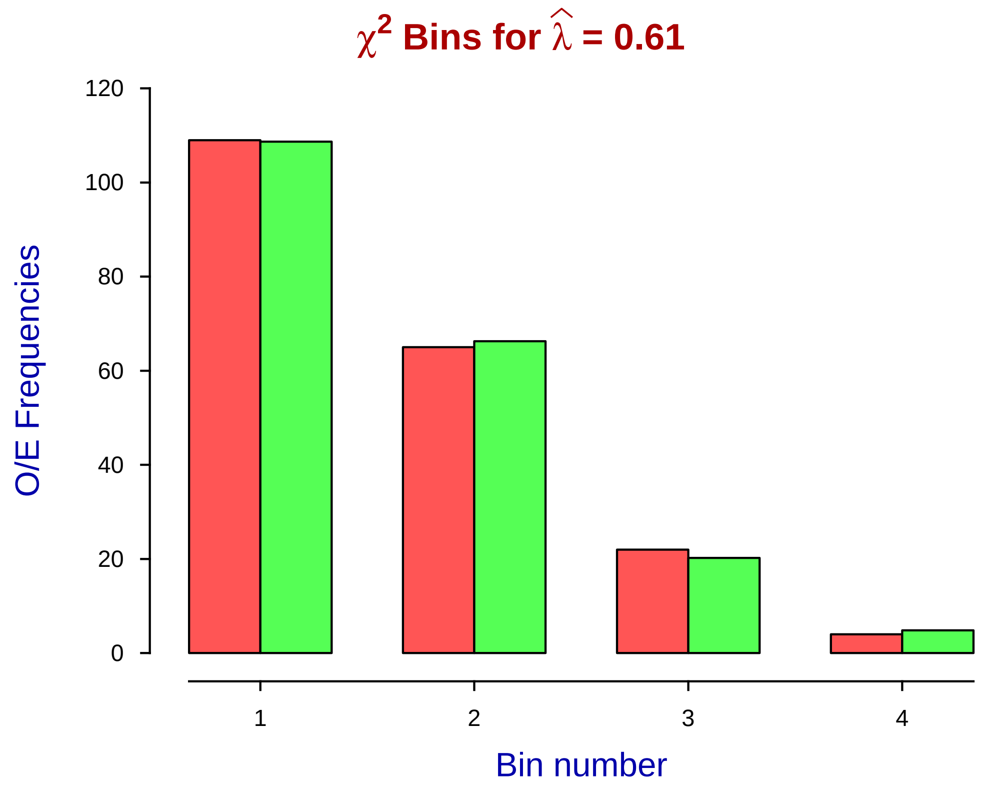
The 200 frequencies recorded are contained in the test file `poisson.tf2` as follows

Deaths	0	1	2	3	4
Frequency	109	65	22	3	1

leading to the result, shown below, that $p = 0.8506$, so that the null hypothesis of a Poisson distribution with $\hat{\lambda} = 0.61$ cannot be rejected.

Chi-square test for $P(\lambda)$ with $\lambda = 0.61$		
H_0: Poisson distribution for data with title:		
Death from horse kicks in Prussian cavalry 1875-1894		
Sample estimate used in chi-square test		
Sample estimate for λ	0.6100	
Lower 95% confidence limit	0.5066	
Upper 95% confidence limit	0.7283	
Mean of x-values	0.6100	
Variance of x-values	0.6110	
Standard deviation of x	0.7816	
Number of partitions (bins) used	4	
Number of degrees of freedom	2	
Chi-square test statistic C	0.3235	
$P(\chi^2 \geq C)$	0.8506	*Consider accepting H_0*
Upper tail 5% critical point	5.991	
Upper tail 1% critical point	9.210	

The observed and expected values used in this test are displayed below.



The Fisher Exact test will frequently fail with large or aberrant data sets and this will be indicated when it happens, but the dispersion test can always be used to test for data that are too uniform or too clustered to be from a Poisson distribution. This is used to study clumping of objects studied microscopically, and similar situations concerning spatial or temporal distributions of counts. Consider, for instance, the analysis of this data set contained in the test file `poisson.tf3`.

Counts	0	1	2	3	4	5	6	7	8	9	10
Frequency	0	2	10	0	0	0	1	2	9	1	5

Dispersion and Fisher-exact Poisson tests

Bonferroni $n = 2$		
Data: Poisson clumping data		
Sample size	30	
Sample total	173	
Sample sum of squares	1333	
Sample mean	5.767	
Lower 95% confidence limit	4.939	
Upper 95% confidence limit	6.693	
Sample variance	11.56	*Too large ?*
Dispersion (D)	58.16	
$P(\chi^2 \geq D)$	0.00104	*Reject H_0 at 1% sig.level*
Number of degrees of freedom	29	
Fisher exact probability	1.00000	
IFAIL = 1: Fisher p is only an upper bound		

In a Poisson distribution the mean is equal to the variance, and a variance much less than the mean suggests a distribution that is too uniform, while a variance exceeding the mean could indicate clustering, as will be clear from the results above and the following plot.