Tutorials and worked examples for simulation, curve fitting, statistical analysis, and plotting.
http://www.simfit.org.uk

The pseudo-inverse (or generalized inverse) of a matrix satisfies many of the properties of the usual inverse in situations when the matrix itself is not square and of full rank. It is encountered in the solution of many rank-deficient data analysis situations, and is best calculated using the singular value decomposition (SVD).

From the main SIMFIT menu choose [Statistics], then [Numerical analysis], and select the pseudo-inverse option which provides the default test file `f01blf.tf1` containing the following matrix.

7	-2	4	9	1.8
3	8	-4	6	1.3
9	6	1	5	2.1
-8	7	5	2	0.6
4	-1	2	8	1.3
1	6	3	-5	0.5

Using the tolerance factor set at $TOL = 1.0E^{-7}$, the rank of this matrix was estimated to be 4 and the following matrix was calculated as the pseudo-inverse.

1.7807132E-02	-1.1826257E-02	4.7156796E-02	-5.6636340E-02	-3.6741218E-03	3.8408070E-02
-2.1564769E-02	4.3417257E-02	2.9445712E-02	2.9132145E-02	-1.3781035E-02	3.4256117E-02
5.2028568E-02	-8.1265321E-02	1.3926152E-02	4.7441829E-02	1.6646584E-02	5.7593532E-02
2.3686052E-02	3.5716849E-02	-1.3808338E-02	3.0477616E-02	3.5665495E-02	-5.7134309E-02
7.1956983E-03	-1.3957472E-03	7.6720321E-03	5.0415250E-03	3.4856923E-03	7.3123409E-03

The estimation of rank is based on the TOL value. The singular values are calculated in descending order and when a value, say the k'th, becomes less than or equal to TOL multiplied by the first (i.e. largest) singular value, than all subsequent singular values are set to zero and the rank is recorded as $k - 1$.

Given the singular value decomposition of a m by n matrix A of rank r

$$A = U\Sigma V^T$$

where $m \geq n \geq r$ the pseudo-inverse is the n by m matrix A^+ satisfying these properties.

1. $AA^+A = A$
2. $A^+AA^+ = A^+$
3. $(A^+A)^T = A^+A$
4. $(AA^+)^T = AA^+$
5. $A^+ = V\Lambda U^T$

If the singular values are $\sigma_1 \geq \ldots \geq \sigma_r > 0$ and $\sigma_i = 0$ for $i > r$, then Λ is found to be

$$\Lambda = \begin{pmatrix} 1/\sigma_1 & 0 & 0 & \ldots & \ldots & \ldots & 0 \\ 0 & 1/\sigma_2 & 0 & \ldots & \ldots & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \ldots & \ldots & \ldots & \ldots & 1/\sigma_r & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & \ldots & \ldots & 0 \end{pmatrix}.$$