*Simfit*

It is frequently required to plot a curve where the model has to be defined parametrically as $x(t), y(t)$ instead of $y = f(x)$, or sometimes a parametric space curve $x(t), y(t), z(t)$ may be required.

Example 1: The curve $r = A\sin(4\theta)$

This is achieved by opening **makdat** then reading in the parametric model file rose_e.mod shown below to create the plot illustrated for the parameter values $p_1 = 1$, and $0 \le \theta \le 2\pi$.

```
%
Example: Eight leaved rose
.............
r = A*sin(4*theta): where theta = x, r = f(1) and A = p(1)
.............
 %
 1 equation
 1 variable
 1 parameter
 %
 begin{expression}
 f(1) = p(1)sin(4x)
 end{expression}
 %
```
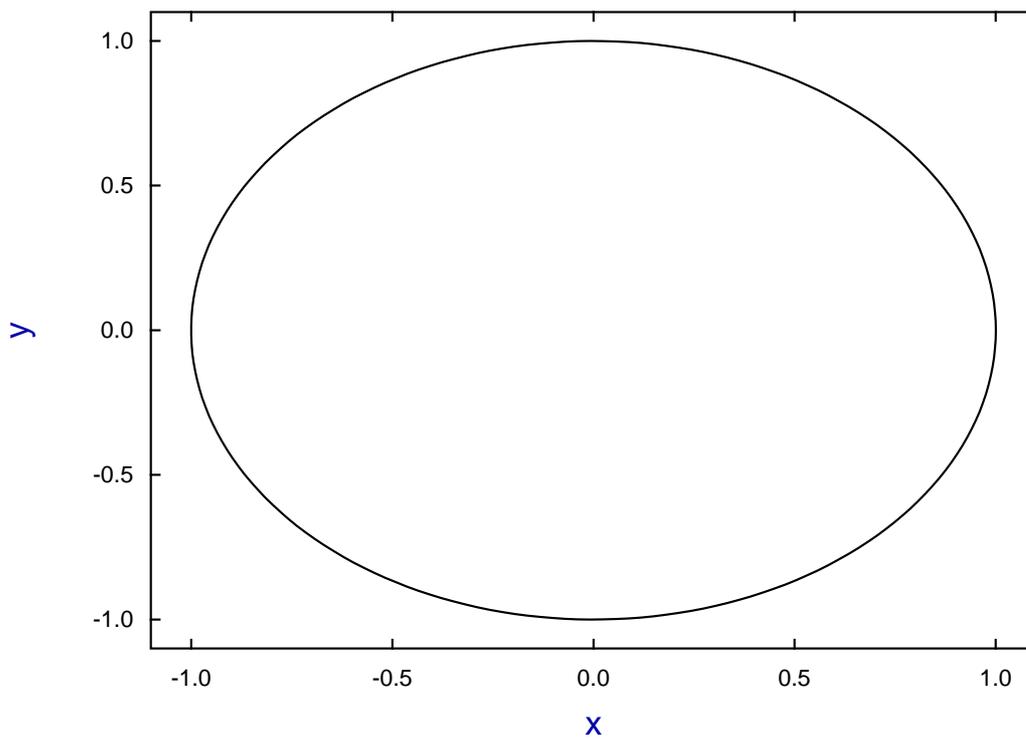
r = sin(4θ): 0 ≤ θ ≤ 2π
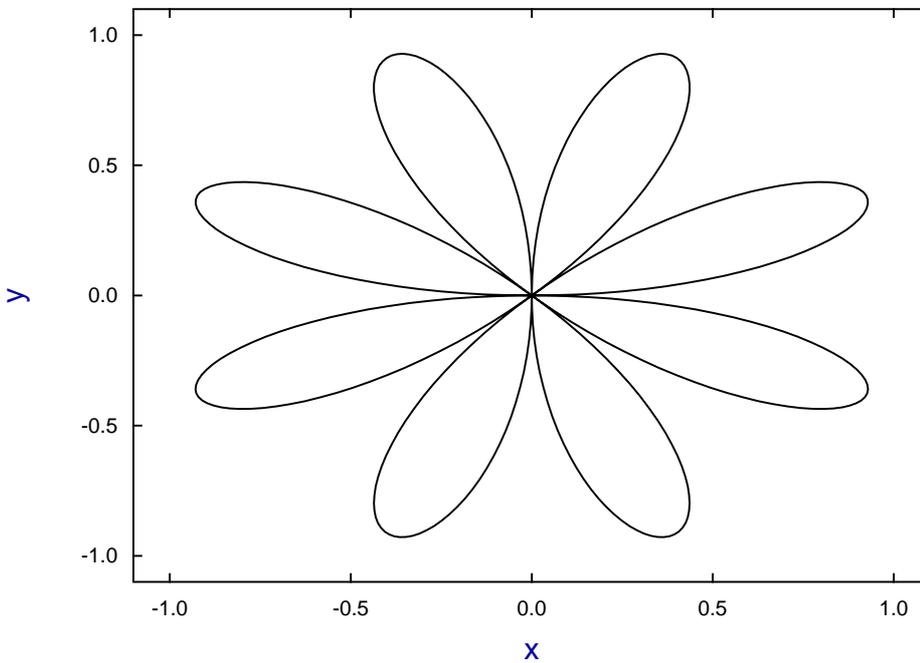
Example 2: The ellipse $x = A\cos(t)$, $y = B\sin(t)$**.**

The following model file `ellipse_e.mod` and graph show how a SIMF I T model file can be used to define
multiple models.

```
 %
Example: the ellipse
        X = A*cos(t), Y = B*sin(t)
        where: t = x, A = p(1), B = p(2)
                and X(t) = f(1), Y(t) = f(2)
 %
 2 equations
 1 variable
 2 parameters
 %
 begin{expression}
 f(1) = p(1)cos(x)
 f(2) = p(2)sin(x)
 end{expression}
 %
```
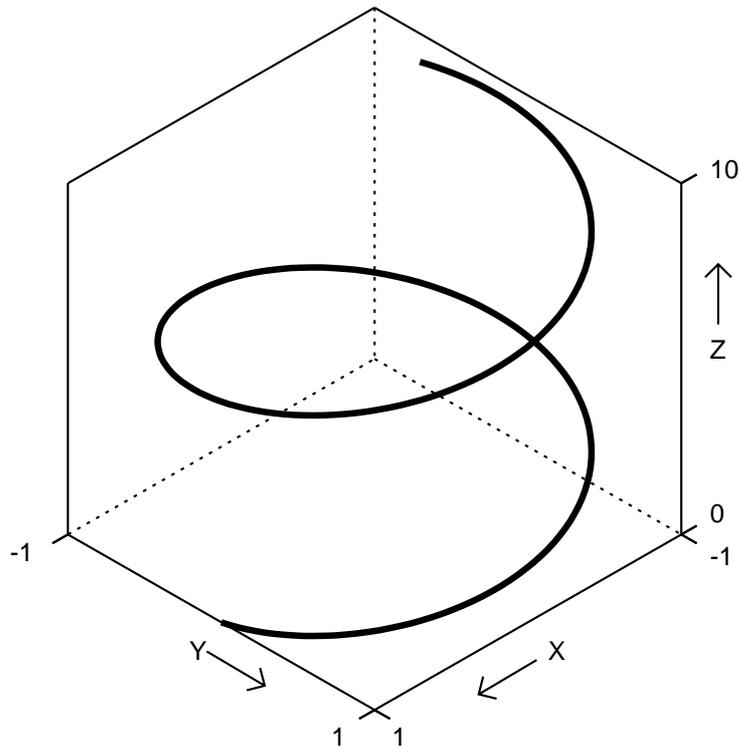

A parametric curve x(t), y(t),

Example 3: The helix $x = A\cos(t)$, $y = B\sin(t)$, $z = Ct$

Model file `helix_e.mod` illustrates how the three coordinates for a space curve can be defined and plotted.

```
%
Example: the helix
        X = A*cos(t), Y = B*sin(t), Z = C*t
        where: t = x, A = p(1), B = p(2), C = p(3)
               and X(t) = f(1), Y(t) = f(2), Z(t) = f(3)
%
3 equations
1 variable
3 parameters
%
begin{expression}
f(1) = p(1)cos(x)
f(2) = p(2)sin(x)
f(3) = p(3)x
end{expression}
%
```


SIMFIT 3D space plot (x,y,z)

## Example 4: Projecting space curves onto planes

Sometimes it is useful to project space curves onto planes for purposes of illustration. The next figure shows a simulation using **usermod** with the model file `twister.mod`.

The parametric equations are

$$x = t\cos t, \; y = t\sin t, \; z = t^2$$

and projections are created by fixing one the variables to a constant value.

Twister Curve with Projections onto Planes
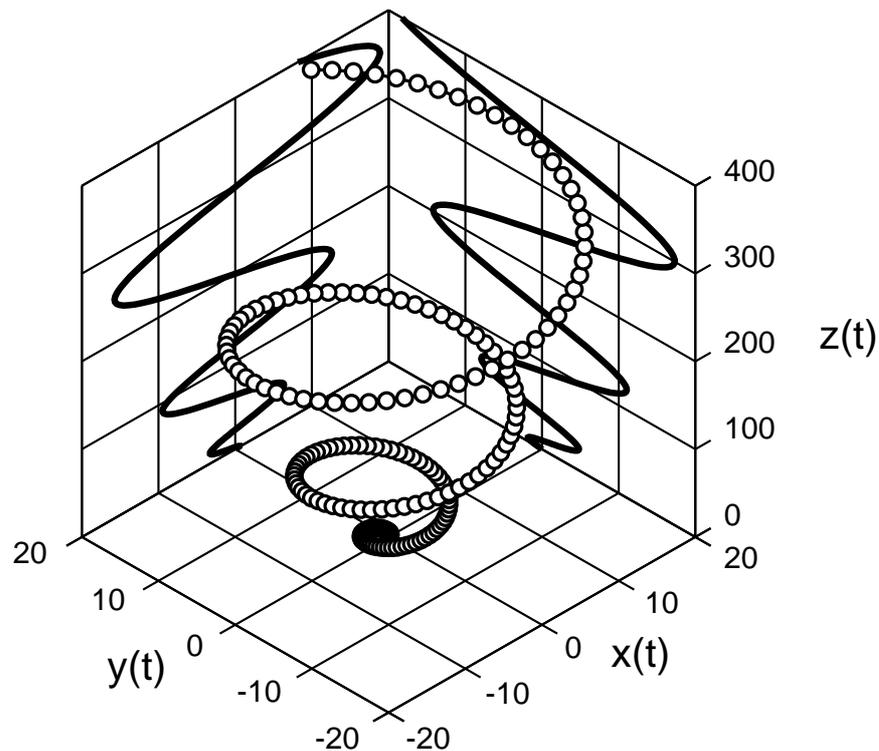
Note the following about the model file `twister.mod`.

- There are 3 curves so there are 9 functions of 1 variable

- The value of x supplied is used as the parameter t

- Functions $f(1)$, $f(4)$, $f(7)$ are the $x(t)$ profiles

- Functions $f(2)$, $f(5)$, $f(8)$ are the $y(t)$ profiles

- Functions $f(3)$, $f(6)$, $f(9)$ are the $z(t)$ profiles

Also observe that the model parameters fix the values of the projection planes just outside the data range, at

$$p(1) = 20, \, p(2) = 20.$$

4

Example 5: Two dimensional families of curves

Users may need to plot families of curves indexed by parameters. For instance, diffusion of a substance from an instantaneous plane source is described by the equation

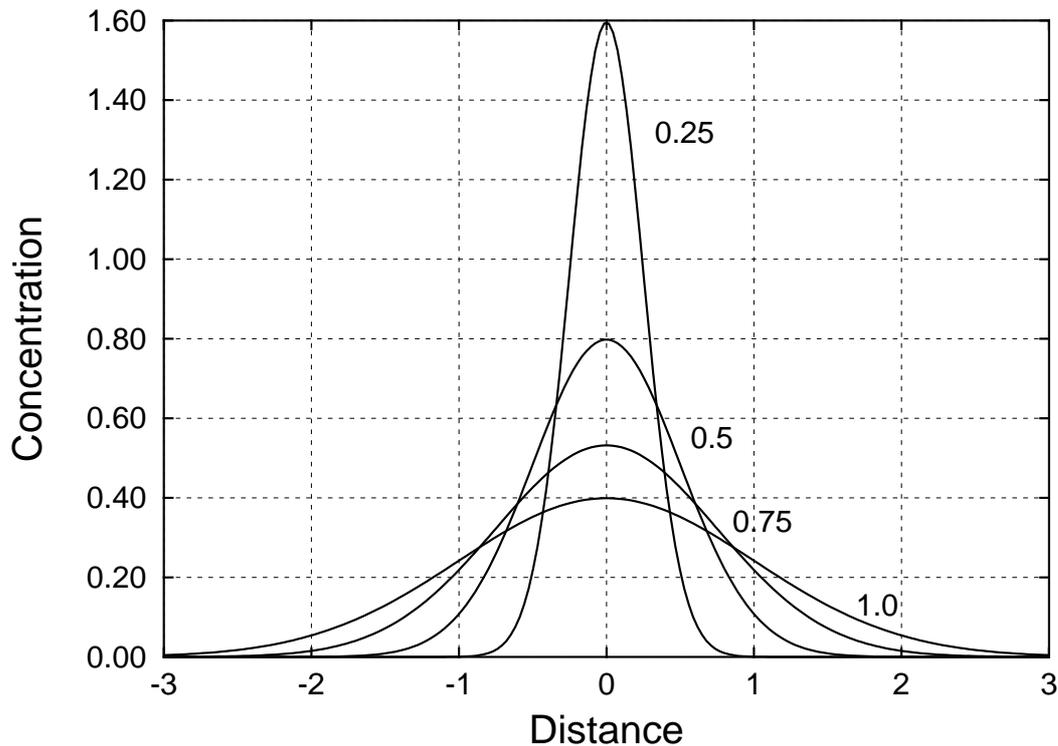$$f(x) = \frac{1}{2\sqrt{\pi Dt}} \exp\left(\frac{-x^2}{4Dt}\right)$$

which is, of course, a normal distribution with $\mu = 0$ and $\sigma^2 = 2Dt$, where D is the diffusion constant and t is time, so that $2Dt$ is the mean square distance diffused by molecules in time t. Now it is easy to plot the concentration $f(x)$ predicted by this equation as a function of distance x and time t given a diffusion constant D, by simulating the equation using **makdat** saving the curves to a library file or project archive, then plotting the collected curves. However, there is a much better way using program **usermod** which has the important advantage that families of curves indexed by parameters can be plotted interactively. This is a more powerful technique which provides numerous advantages and convenient options when simulating systems to observe the behavior of the profiles as the indexing parameters vary.

The next figure shows the above equation plotted (in arbitrary units) using the model parameters

$$p_i = 2Dt_i, \text{ for } i = 1, 2, 3, 4$$

to display the diffusion profiles as a function of time. The plot was created using the model file `family2d.mod`, which simply defines four identical equations corresponding to the diffusion equation but with four different parameters p_i. Program **usermod** was then used to read in the model, simulate it for the parameter values indicated, then plot the curves simultaneously.

Example 6: Three dimensional families of curves

Users may need to plot families of curves indexed by parameters in three dimensions. To show how this is done, the diffusion equation dealt with previously in Example 5 is reformulated, using $y = \sqrt{2Dt}$, as

$$z(x, y) = \frac{1}{y\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{x}{y}\right)^2\right\}$$

and is plotted in the next figure for the same parameter values used before, but now as sections through the surface of a function of two variables.

Diffusion From a Plane Source
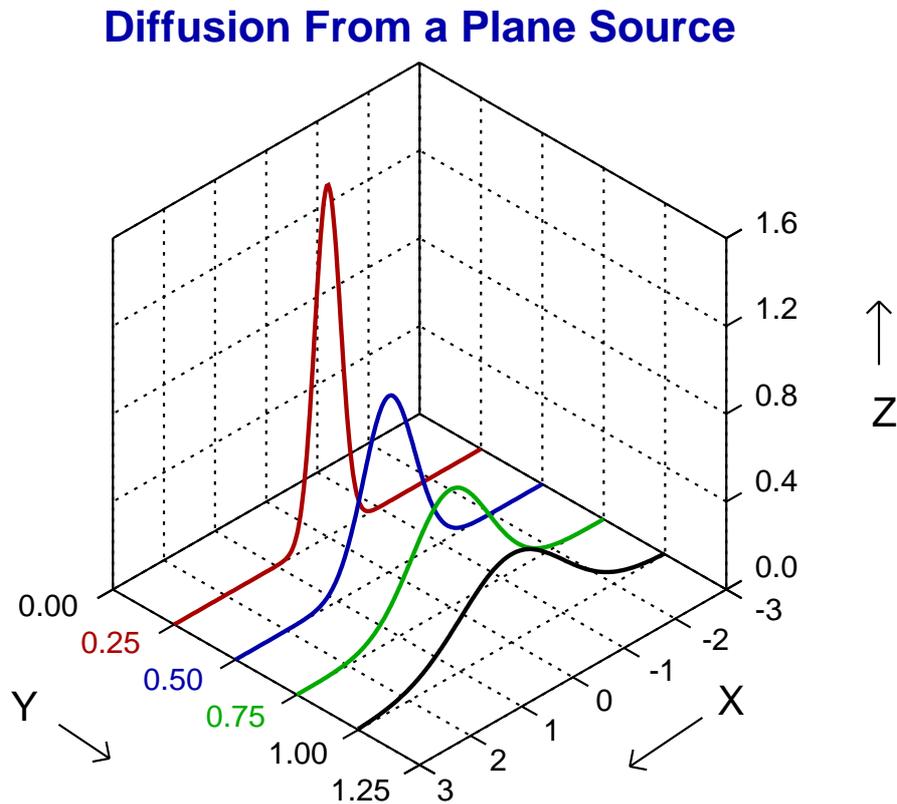
This is, of course, a case of a family of parametric space curves projected onto the fixed values of y. Now the model file `family3d.mod` was used by program **usermod** to create this figure, using the option to plot n sets of parametric space curves, but you should observe a number of important facts about this model file before attempting to plot your own families of space curves.

- There are 4 curves so there are 12 functions of 1 variable

- Functions $f(1)$, $f(4)$, $f(7)$, $f(10)$ are the parameter t, i.e. x

- Functions $f(2)$, $f(5)$, $f(8)$, $f(11)$ are the y values, i.e. $\sqrt{2Dt}$

- Functions $f(3)$, $f(6)$, $f(9)$, $f(12)$ are the z values, i.e. the concentration profiles

Finally, it is clear that n space curves require a model file that specifies $3n$ equations, but you should also realize that space curves cannot be plotted if there is insufficient variation in any of the independent variables, e.g. if all $y = k$, for some fixed parameter k.