



Tutorials and worked examples for simulation,  
curve fitting, statistical analysis, and plotting.  
<http://www.simfit.org.uk>

The unpaired  $t$  test is used to see if it is reasonable to conclude that two sets of independent observations have the same population means. It is based on the assumptions that

- Both samples are normally distributed
- Both samples have the same variance
- Both sample sizes are greater than 1 (and preferably very much greater)

and hence it is equivalent to analysis of variance (ANOVA) with just two columns.

To be precise, the user has two samples (i.e. vectors  $X$  and  $Y$ ) with  $m$  and  $n$  observations

$$X = (x_1, x_2, \dots, x_m)$$

$$Y = (y_1, y_2, \dots, y_n)$$

and wishes to test the null hypothesis that the samples have the same population means,  $\mu_x$  and  $\mu_y$ , against the alternative hypothesis that they are not equal, or possibly the one-sided alternatives. That is

$$H_0 : \mu_x = \mu_y$$

$$H_1 : \mu_x \neq \mu_y$$

$$H_2 : \mu_x > \mu_y$$

$$H_3 : \mu_x < \mu_y$$

and SIMFIT provides all the information that is required to perform such tests.

From the main SIMFIT menu select [ A / Z ], choose to open program **ttest**, then analyze the test files provided.

The first choice offered is to test for a normal distribution and equal variances and, if these are chosen, we get the following analysis.

#### Normal distribution test 1

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Data:  $X$ -data for  $t$  test

Shapiro-Wilks statistic  $W$  0.9539

Significance level for  $W$  0.7146 *Tentatively accept normality*

#### Normal distribution test 2

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Data:  $Y$ -data for  $t$  test

Shapiro-Wilks statistic  $W$  0.9360

Significance level for  $W$  0.5089 *Tentatively accept normality*

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This informs us that the Shapiro-Wilks test does not provide any evidence to reject the assumption that both samples are normally distributed. However, this test should only be used if both  $m$  and  $n$  are sufficiently large, say  $m, n > 20$ , so in this case we should really have chosen not to do Shapiro-Wilks tests.

Then a  $F$  test for equality of variances is carried out, yielding the next results.

### **F test for equality of variances**

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|   |        |
|---|--------|
| X-data: Test file <code>ttest.tf2</code> : for paired $t$ test with TTEST.TF3 |        |
| Number of $x$ -values   | 10     |
| Mean $x$  | 14.80  |
| Sample variance of $x$  | 21.96  |
| Sample standard deviation of $x$  | 4.686  |
| Y-data: Test file <code>ttest.tf3</code> : for paired $t$ test with TTEST.TF2 |        |
| Number of $y$ -values   | 10     |
| Mean $y$  | 16.10  |
| Sample variance of $y$  | 24.54  |
| Sample standard deviation of $y$  | 4.954  |
| Variance ratio $VR$   | 1.118  |
| Degrees of freedom (numerator)  | 9      |
| Degrees of freedom (denominator)  | 9      |
| $P(F \geq VR)$  | 0.4354 |
| Two tail $p$ value  | 0.8708 |
| Conclusion: <i>Consider accepting equality of variances</i>                   |        |

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Again, although this does not reject equality of variances, it should be pointed out that this test is only reliable with large samples, say  $m, n > 50$  and should not have been performed with such small samples. It is anticipated that, for routine analysis with small samples, the Shapiro-Wilks and variance ratio tests would be switched off.

Finally, the unpaired  $t$  test yields the following results.

### **Unpaired $t$ test ([ ] = corrected for unequal variances)**

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|   |         |            |
|---|---------|------------|
| Number of $x$ -values                                   | 10      |            |
| Number of $y$ -values                                   | 10      |            |
| Number of degrees of freedom                            | 18      | [ 18]      |
| Unpaired $t$ test statistic $U$                         | -0.6029 | [ -0.6029] |
| $P(t \geq U)$ (upper tail $p$ )                         | 0.7229  | [ 0.7229]  |
| $P(t \leq U)$ (lower tail $p$ )                         | 0.2771  | [ 0.2771]  |
| $p$ for two tailed $t$ test                             | 0.5541  | [ 0.5541]  |
| Difference between means $DM$                           | -1.300  |            |
| Lower 95% confidence limit for $DM$                     | -5.830  | [ -5.830]  |
| Upper 95% confidence limit for $DM$                     | 3.230   | [ 3.230]   |
| Conclusion: <i>Consider accepting equality of means</i> |         |            |

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Note that  $p$  values are given for either two-tailed or one-tailed testing, but this is just for convenience as users should have decided in advance which  $p$  value to accept, or in doubt would usually just rely on the two-tailed test.

For situations where there is doubt about variance equality, corrected values are given, but in this case where the sample size is small and the data are actually paired, correction is not required. Provided that deviations from normality and variance equality are fairly small, the unpaired  $t$  test has been claimed to be reassuringly robust. However this does not mean that using sample sizes much less than 10 is acceptable.

To explore these last points note that SIMFIT can calculate power as a function of sample size and, in addition, has extensive facilities to explore particular situations concerning sample size, deviations from normality, and variance inequality by simulation.